

**Attendance Problems. Find the image point when the indicated transformation is applied to the given pre-image point.**

1.  $(x,y) \rightarrow (x+3,y-1); (2,4)$

2.  $(x,y) \rightarrow (x,-y); (-2,1)$

3.  $(x,y) \rightarrow (3x,3y); (-5,0)$

4.  $(x,y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right); (-3,6)$

- I can draw and describe similarity transformations in the coordinate plane.
- I can use properties of similarity transformations to determine whether polygons are similar and to prove circles similar.

**Vocabulary:** Similarity Transformation

### **Common Core**

**CC.9-12.G.C.1** Prove that all circles are similar.

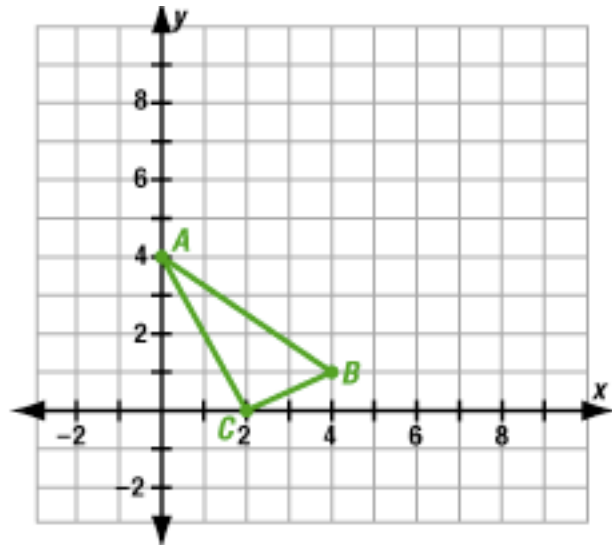
**CC.9-12.G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor:

- A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

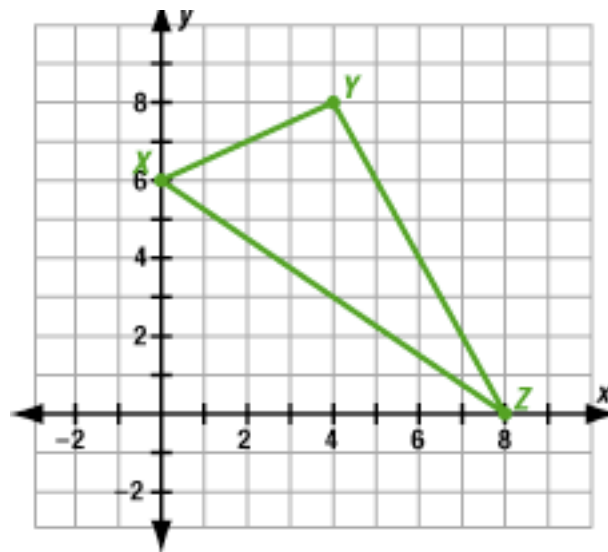
A transformation that produces similar figures is a *similarity transformation*. A **similarity transformation** is a dilation or a composite of one or more dilations and one or more congruence transformations. Two figures are similar if and only if there is a similarity transformation that maps one figure to the other figure.

**Refer to video example 1.**

- A.  $D:(x,y) \rightarrow (2x,2y)$   
 $A(0,4), B(4,1) C(2,0)$



B.  $D(x,y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$   
 $X(0,6), Y(4,8), Z(8,0)$

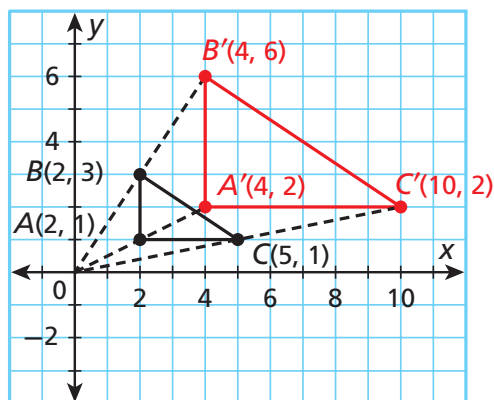


## 1 Drawing and Describing Dilations

Apply the dilation  $D$  to the polygon with the given vertices. Describe the dilation.

**A**  $D: (x, y) \rightarrow (2x, 2y)$

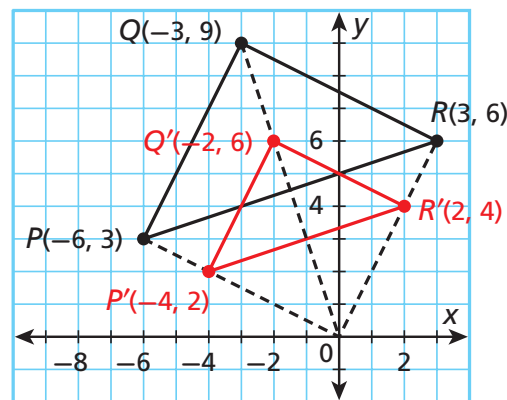
$A(2, 1), B(2, 3), C(5, 1)$



This is a dilation with center  $(0, 0)$  and scale factor 2.

**B**  $D: (x, y) \rightarrow \left(\frac{2}{3}x, \frac{2}{3}y\right)$

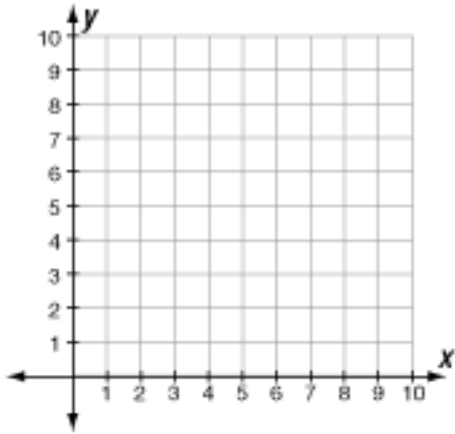
$P(-6, 3), Q(-3, 9), R(3, 6)$



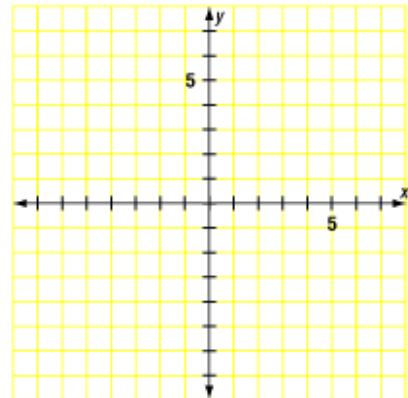
This is a dilation with center  $(0, 0)$  and scale factor  $\frac{2}{3}$ .

**Example 1.** Apply the dilation **D** to the polygon with the given vertices.  
**Describe the dilation.**

**A.**  $D:(x,y) \rightarrow (3x, 3y)$   
 $A(1,1), B(3,1), C(3,2)$



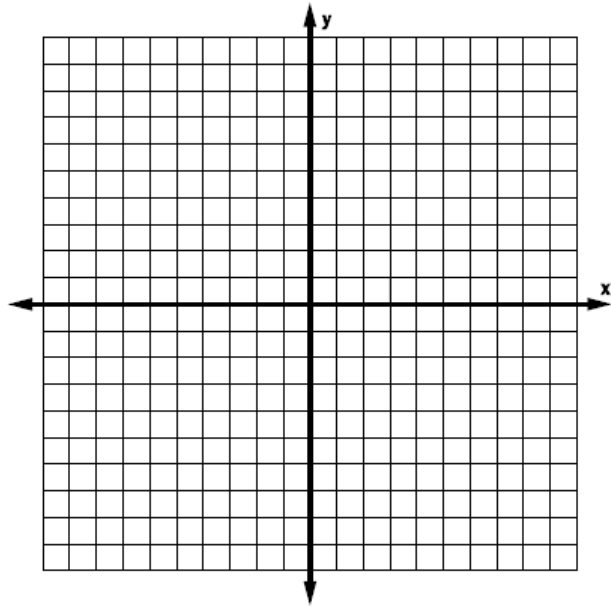
**B.**  $D:(x,y) \rightarrow \left(\frac{3}{4}x, \frac{3}{4}y\right)$   
 $P(-8,4), Q(-4,8), R(4,4)$



**5. Guided Practice:** Apply the dilation  $D$  to the polygon with the given vertices. Name the coordinates of the image points. Identify and describe the transformation.

$$D: (x, y) \rightarrow \left(\frac{3}{4}x, \frac{3}{4}y\right)$$

$A(4, 8)$ ,  $B(-8, 4)$ ,  $C(8, -4)$



### Remember!

Translations, reflections, and rotations are congruence transformations.

**Circle 1:** I am so enlarged lately that I don't even want to tell you my scale factor!

**Circle 2:** Don't worry about it. All circles are similar!

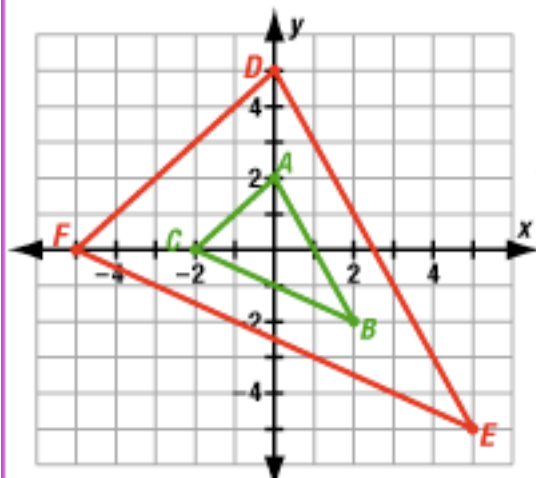
"A life is not important except in the impact it has on other lives."

*Baseball Player, Jackie Robinson*

**Refer to video example 2.**

**A.**

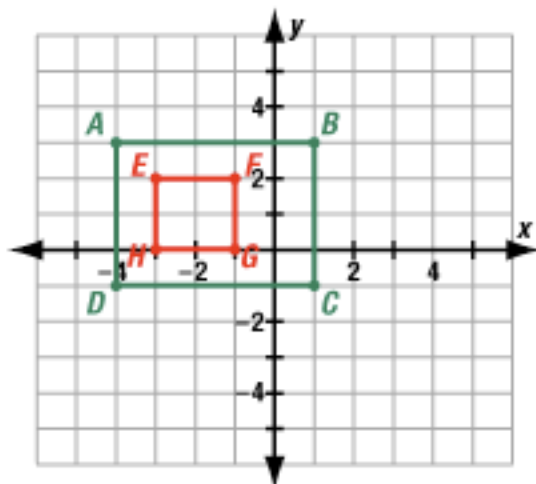
$A(0, 2)$ ,  $B(2, -2)$ ,  $C(-2, 0)$   
 $D(0, 5)$ ,  $E(5, -5)$ ,  $F(-5, 0)$



$\triangle ABC: A(0, 2), B(2, -2), C(-2, 0)$

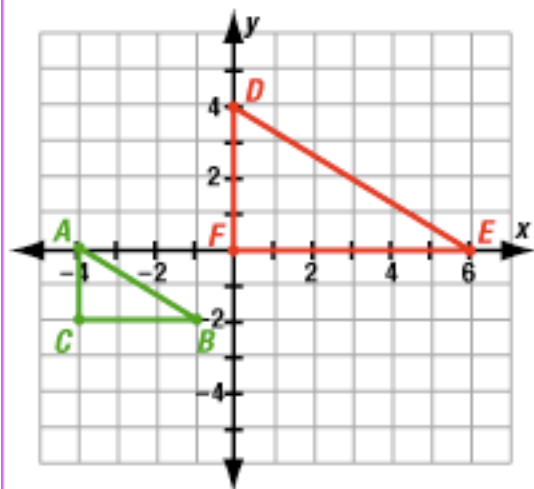
**B.**

$A(-4, 3)$ ,  $B(1, 3)$ ,  $C(1, -1)$ ,  $D(-4, -1)$   
 $E(-3, 2)$ ,  $F(-1, 2)$ ,  $G(-1, 0)$ ,  $H(-3, 0)$



C.

$A(-4, 0)$ ,  $B(-1, -2)$ ,  $C(-4, -2)$   
 $D(0, 4)$ ,  $E(6, 0)$ ,  $F(0, 0)$



$\triangle ABC: A(-4, 0), B(-1, -2), C(-4, -2)$

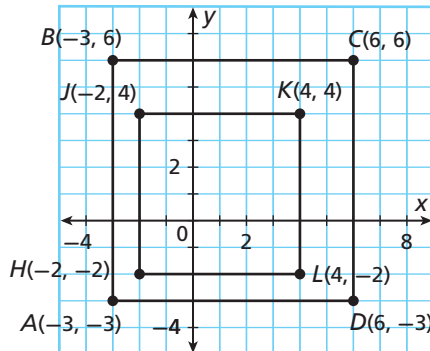
## 2 Determining Whether Polygons are Similar

Determine whether the polygons with the given vertices are similar.

- A**  $A(-3, -3)$ ,  $B(-3, 6)$ ,  $C(6, 6)$ ,  $D(6, -3)$   
 $H(-2, -2)$ ,  $J(-2, 4)$ ,  $K(4, 4)$ ,  $L(4, -2)$

Yes;  $ABCD$  can be mapped to  $HJKL$

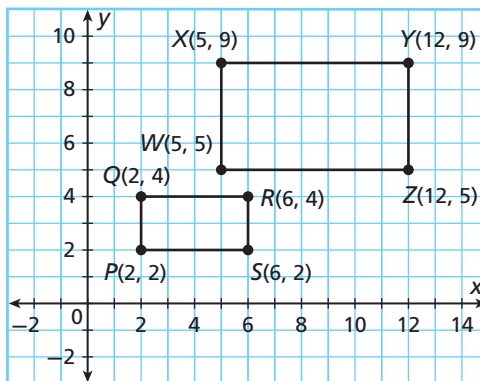
by a dilation:  $(x, y) \rightarrow \left(\frac{2}{3}x, \frac{2}{3}y\right)$ .



- B**  $P(2, 2)$ ,  $Q(2, 4)$ ,  $R(6, 4)$ ,  $S(6, 2)$   
 $W(5, 5)$ ,  $X(5, 9)$ ,  $Y(12, 9)$ ,  $Z(12, 5)$

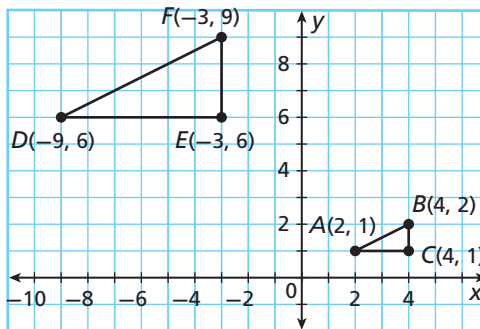
No;

The rule  $(x, y) \rightarrow (2.5x, 2.5y)$  maps  $P$  to  $W$ , but not  $Q$  to  $X$ . No similarity transformation maps  $PQRS$  to  $WXYZ$ .

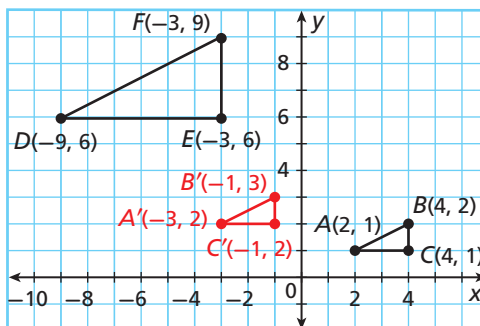


- C**  $A(2, 1)$ ,  $B(4, 2)$ ,  $C(4, 1)$   
 $D(-9, 6)$ ,  $E(-3, 6)$ ,  $F(-3, 9)$

Yes; Translate  $\triangle ABC$  to the left and up. Then enlarge the image to obtain  $\triangle DEF$ .



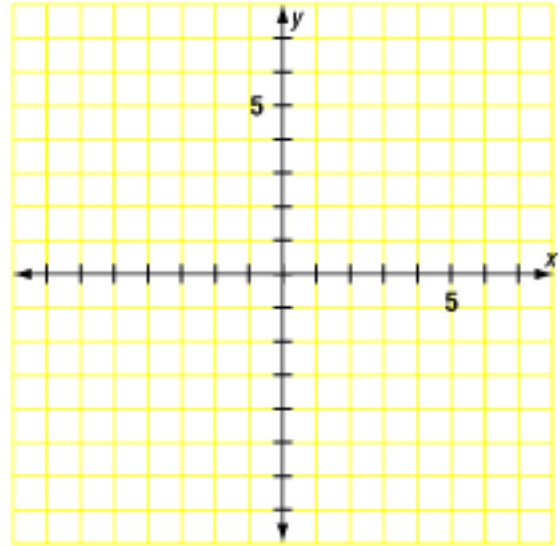
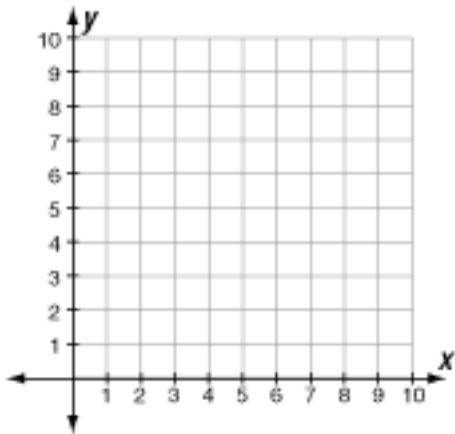
Yes;  $\triangle ABC$  can be mapped to  $\triangle A'B'C'$  by a translation:  
 $(x, y) \rightarrow (x - 5, y + 1)$ . Then  
 $\triangle A'B'C'$  can be mapped to  $\triangle DEF$  by a dilation:  
 $(x, y) \rightarrow (3x, 3y)$ .



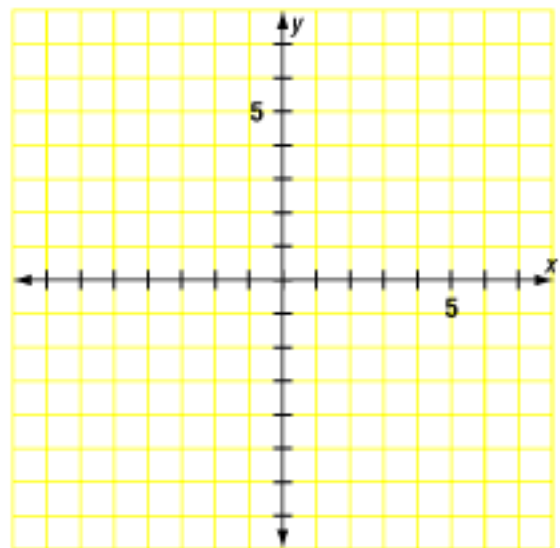
**Example 2.** Determine whether the polygons with the given vertices are similar.

**A.**  $A(-6, -6)$ ,  $B(-6, 3)$ ,  $C(3, 3)$ ,  $D(3, -6)$  and  $H(-2, -2)$ ,  $J(-2, 1)$ ,  $K(1, 1)$ ,  $L(1, -2)$

**B.**  $P(2, 0)$ ,  $Q(2, 4)$ ,  $R(4, 4)$ ,  $S(4, 0)$  and  $W(5, 0)$ ,  $X(5, 10)$ ,  $Y(8, 10)$ ,  $Z(8, 0)$ .



**C.**  $A(1, 2)$ ,  $B(2, 2)$ ,  $C(1, 4)$  and  $D(4, -6)$ ,  $E(6, -6)$ ,  $F(4, -2)$



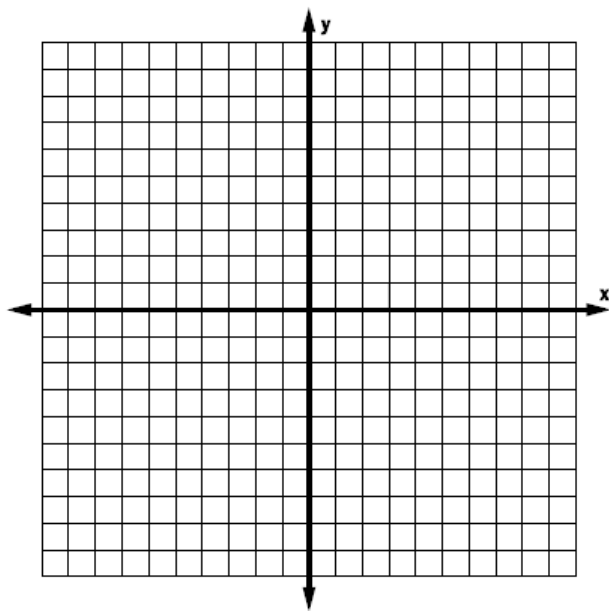
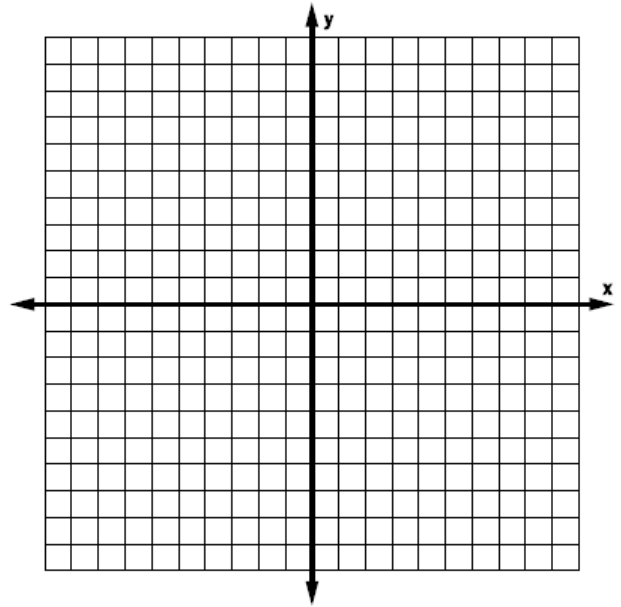


**D.**  $F(3, 3)$ ,  $G(3, 6)$ ,  $H(9, 3)$ ,  $J(9, -3)$  and  $S(-1, 1)$ ,  $T(-1, 2)$ ,  $U(-3, 1)$ ,  $V(-3, -1)$ .

**Guided Practice. Determine whether the polygons with the given vertices are similar. Support your answer by describing a transformation.**

$W(-4, 2)$ ,  $X(-4, 6)$ ,  $Y(6, 2)$ ,  $Z(6, 6)$

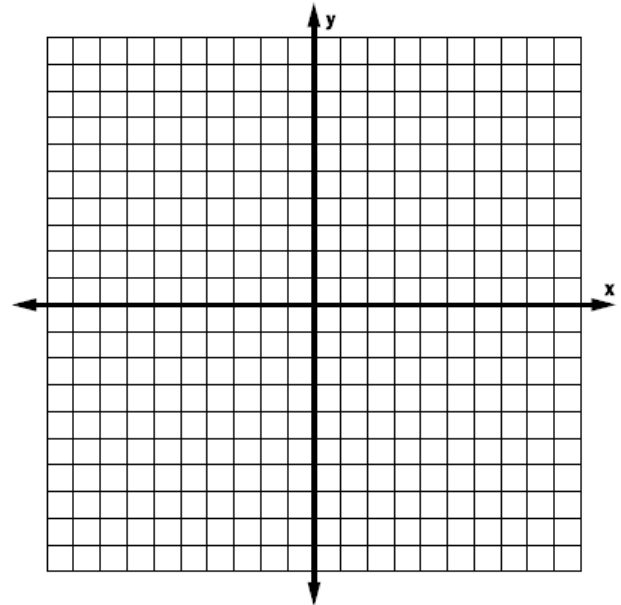
**6.**  $D(-2, 1)$ ,  $E(-8, 12)$ ,  $F(3, 1)$ ,  $G(3, 3)$



7.

$L(-10, 5)$ ,  $M(-5, 0)$ ,  $N(0, 0)$ ,  $O(5, 5)$

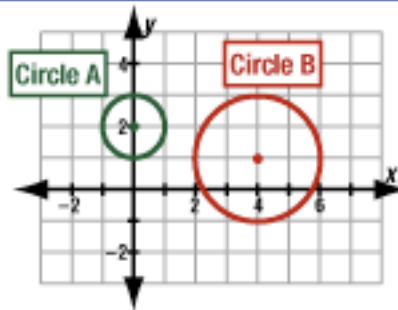
$D(4, 2)$ ,  $E(2, 0)$ ,  $F(0, 0)$ ,  $G(-2, 2)$



## 7-2 Similarity and Transformations (p 477) 14-18.

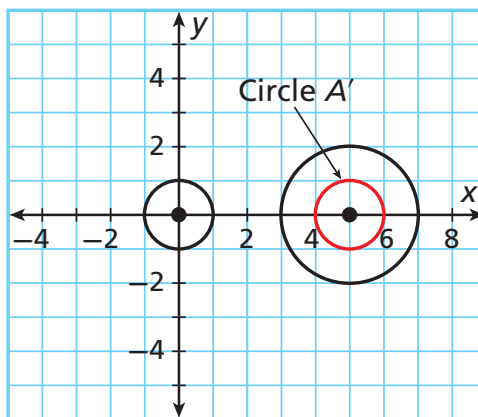
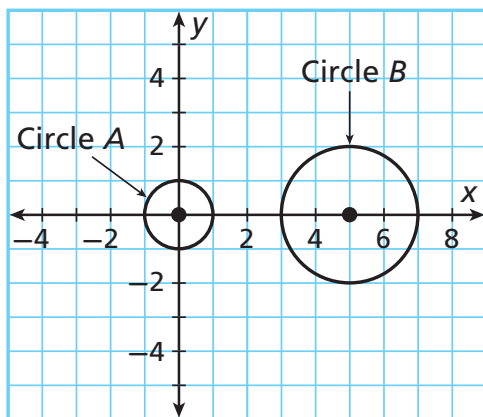
Refer to video example 3.

Prove that circle A with center  $(0, 2)$  and radius 1 is similar to circle B with center  $(4, 1)$  and radius 2.



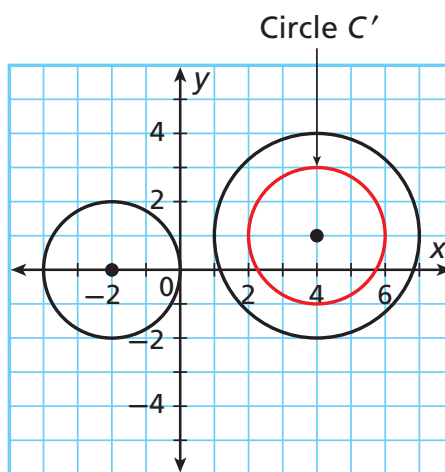
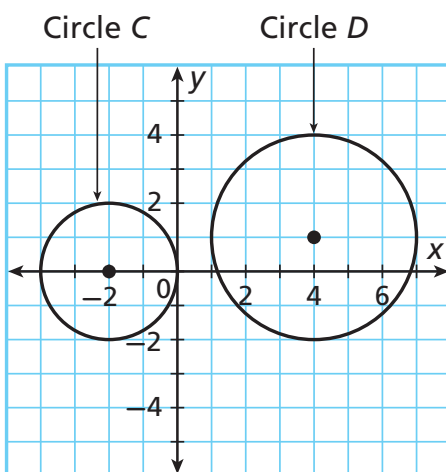
### 3 Proving Circles Similar

- A** Prove that circle  $A$  with center  $(0, 0)$  and radius 1 is similar to circle  $B$  with center  $(5, 0)$  and radius 2.



Circle  $A$  can be mapped to circle  $A'$  by a translation:  $(x, y) \rightarrow (x + 5, y)$ . Circle  $A'$  and circle  $B$  both have center  $(5, 0)$ . Then circle  $A'$  can be mapped to circle  $B$  by a dilation with center  $(5, 0)$  and scale factor 2. So circles  $A$  and  $B$  are similar.

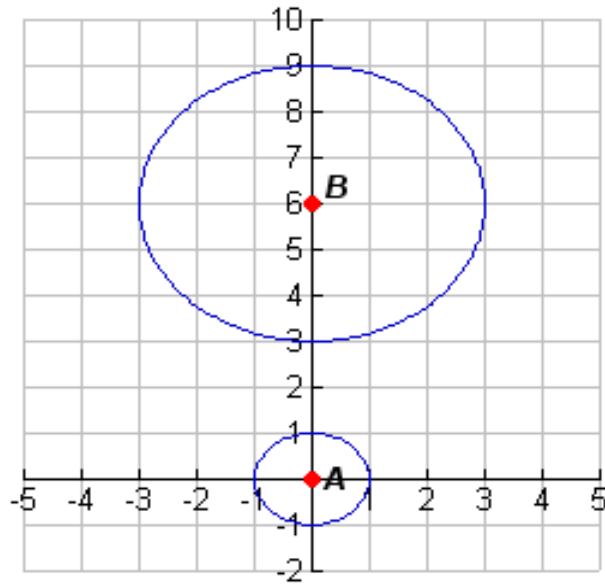
- B** Prove that circle  $C$  with center  $(-2, 0)$  and radius 2 is similar to circle  $D$  with center  $(4, 1)$  and radius 3.



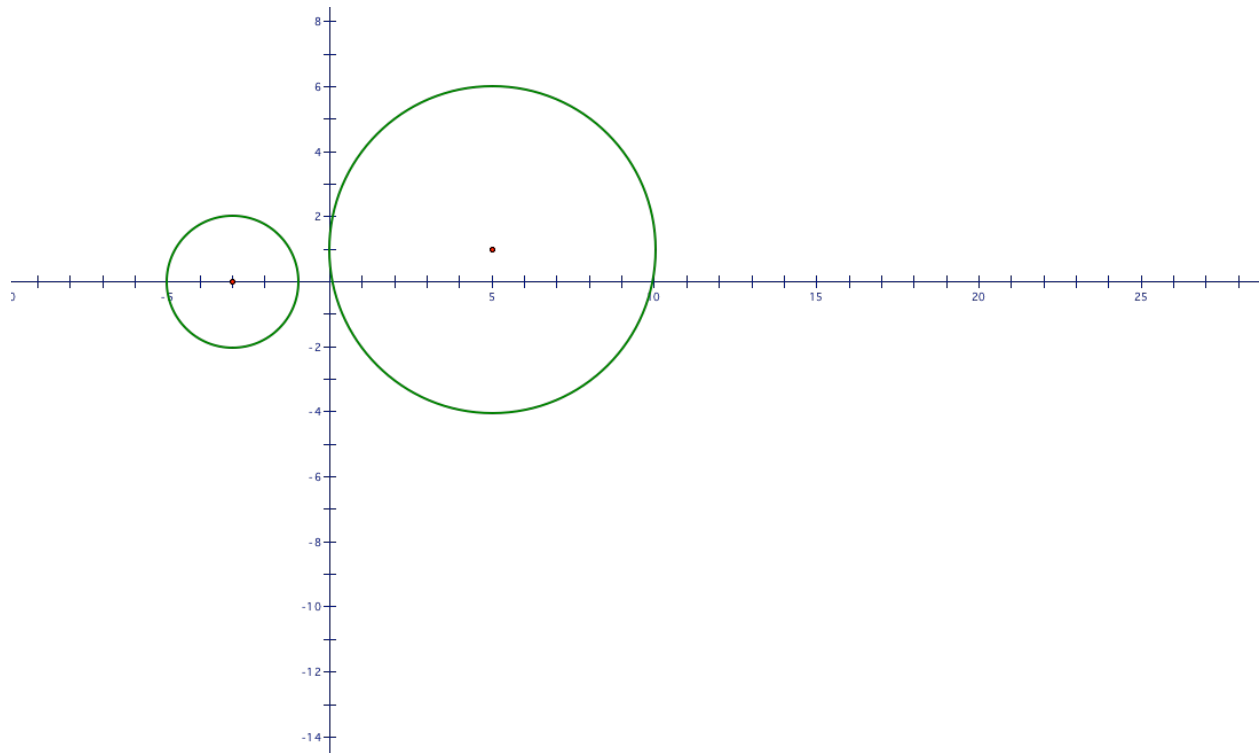
Circle  $C$  can be mapped to circle  $C'$  by a translation:  $(x, y) \rightarrow (x + 6, y + 1)$ . Circle  $C'$  and circle  $D$  both have center  $(4, 1)$ . Then circle  $C'$  can be mapped to circle  $D$  by a dilation with center  $(4, 1)$  and scale factor  $\frac{3}{2}$ . So circles  $C$  and  $D$  are similar.

**Example 3.**

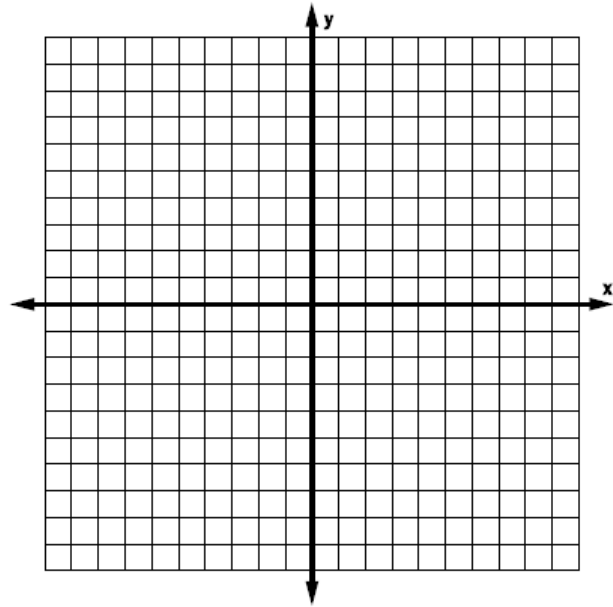
**A.** Prove that circle *A* with center  $(0, 0)$  and radius 1 is similar to circle *B* with center  $(0, 6)$  and radius 3.



**B.** Prove that Circle *C* with center  $(0, -3)$  and radius 2 is similar to circle *D* with center  $(5, 1)$  and radius 5.



**8. Guided Practice:** Prove that circle A with center (6, -9) and radius 4 is similar to circle B with center (3, -8) and radius 5.



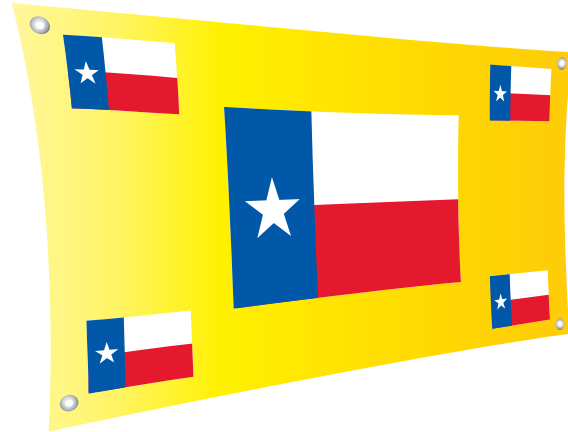
Refer to video example 4.

Prof. Burger is making a sign that shows five rectangular math logos, where the height of each logo is  $\frac{1}{2}$  the length. The middle logo is three times the size of each of the other logos. He will first draw the lower left logo, then the upper left and middle logos. How can he draw those logos?



**4 Business Application**

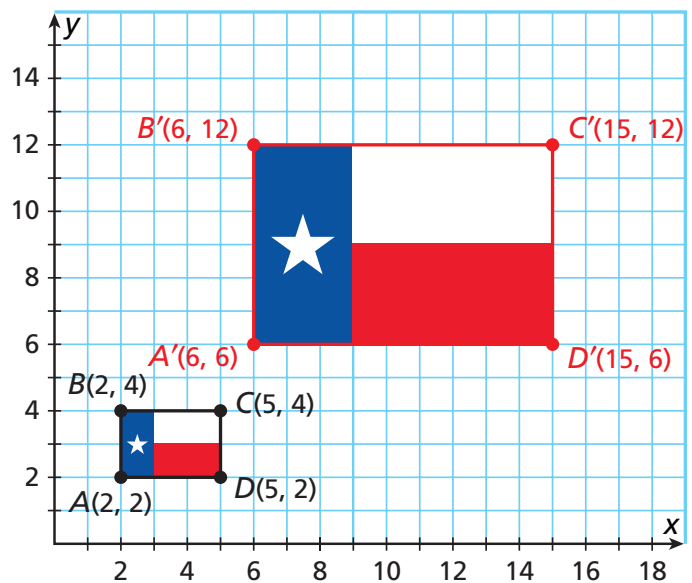
Tia makes signs and banners. She is making a banner that shows five Texas flags. The middle flag is 3 times the size of each of the other flags. Tia will first draw the lower left flag and then the middle flag. How can she draw those flags?



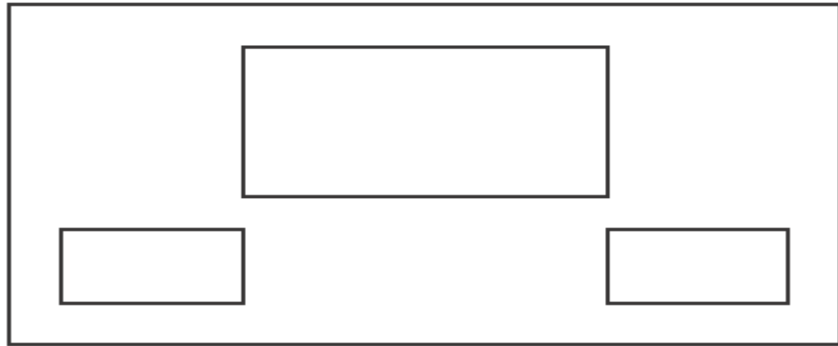
Place the lower left flag on a coordinate plane in a convenient position, such as that shown by rectangle  $ABCD$ .

Apply the dilation with center  $(0, 0)$  and scale factor 3:  $(x, y) \rightarrow (3x, 3y)$ .

The image,  $A'B'C'D'$ , represents the middle flag.



**Example 4.** Eric wants to make a drawing to show how he will arrange three rugs in his room. The large rug is twice the size of each small rug. Eric will first draw the small rug in the lower left corner and then the large rug in the middle. How can he draw those rugs?



**12.** Hector is making an art project by cutting and gluing shapes to a wooden board. His design includes two similar triangles, with one 4 times the size of the other. He cuts and traces the small triangle first onto grid paper. Describe how he can use the tracing to make a pattern for the large fabric triangle.

**7-2 Similarity and Transformations** (p 477) 14-18, 21-24.

